Exam. Code : 211002
Subject Code : 4278

M.Sc. (Mathematics) 2nd Semester

MATH-565 : DIFFERENTIAL AND INTEGRAL EQUATIONS

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt any ***five*** questions by selecting at least ***one*** question from each section. The **fifth** question may be attempted from any section.

SECTION—A

1. (a) Find the general integral of $2x(y+z^2)p+y(2y+z^2)q = z^3$. 8

   (b) Find a complete integral of $f = (p^2 + q^2)y - qz = 0$, using Charpit's method. 12

2. (a) By Jacobi's method, solve the equation $p^2x + q^2y = z$. 12

   (b) Find a complete integral of $zpq - p - q = 0$. 8

SECTION—B

3. Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, $0 < x < 1, t > 0$ subject to the conditions $y(0, t) = y(1, t) = 0$; $y(x, 0) = x(1 - x)$, $y_t(x, 0) = 0$, $0 \le x \le 1$. 20

4. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $-\infty < x < \infty$, $y > 0$ subject to the conditions $u(x, 0) = f(x)$, $-\infty < x < \infty$, u is bounded as $y \to \infty$, u and u_x vanish as $|x| \to \infty$. 20

SECTION—C

5. Solve the Volterra integral equation $y(x) = 1 + \int_0^x xt\, y(t)dt$. 20

6. Using the method of successive approximations, solve the Volterra integral equation :

$$y(x) = 1 + x - \int_0^x y(t)\, dt,\ y_0(x) = 1.$$ 20

SECTION—D

7. Solve the Fredholm's integral equation :

$$y(x) = 2x - \pi + 4 \int_0^{\pi/2} \sin^2 xy(t)\, dt.$$ 20

8. Solve the Fredholm's integral equation, using the method of successive approximations :

$$y(x) = f(x) + \lambda \int_0^1 e^{x-t}\, y(t)dt.$$ 20